

ALGEBRA

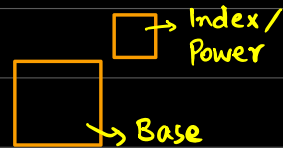
(20-25 Marks)

- 1- Indices
 - 2- Factorization
 - 3- Quadratic Equation Solving.
 - 4- Making Subject.
 - 5- Simultaneous Eq.
 - 6- Functions.
 - 7- Algebraic Expansions.
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ALGEBRA

INDICES

INDEX = POWER
INDICES = POWERS



$$\boxed{1} \quad a^m \times a^n = a^{m+n}$$

(Same bases) (Multiply) = Powers added.

$$x^7 \times x^{12} = x^{19}$$

$$a^m \div a^n = a^{m-n}$$

(Same bases) (Divide) = Powers Subtract.

$$10^{12} \div 10^4 = 10^8$$

CAUTION

$$a^m + a^n = \times$$

$$a^m - a^n = \times$$

Cannot be
further
simplified.

GOLDEN RULE: ALL PROPERTIES OF
INDICES ONLY WORK
FOR MULTIPLY / DIVIDE

Example:

$$1) 2^3 \times 2^5 = 2^{3+5} = 2^8$$

$$2) \frac{4^5}{4^2} = 4^{5-2} = 4^3$$

$$\boxed{2} a^m \times b^m = (a \times b)^m$$

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

If two different bases with same power are being multiplied/divided you can take power common

CAUTION

You are not allowed to take power common if terms are being add/sub

$$a^m + b^m = \cancel{(a+b)^m}$$

$$a^m - b^m = \cancel{(a-b)^m}$$

$$a^2 + b^2 = \cancel{(a+b)^2}$$

$$a^2 - b^2 = \cancel{(a-b)^2}$$

$$a^2 \times b^2 = (a \times b)^2 \checkmark$$

$$\boxed{3} \quad (a \times b)^m = a^m \times b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

You are allowed to open power directly if terms are being Multiplied/Divided.

CAUTION:

YOU ARE NOT ALLOWED TO OPEN POWER DIRECTLY IF TERMS ARE BEING ADD/SUB

$$(a+b)^m = \cancel{a^m + b^m}$$

$$(a-b)^m = \cancel{a^m - b^m}$$

$$(x+4)^2 = \cancel{x^2 + 4^2} \quad \cancel{x^2 + 16}$$

$$(x+2)^2 - 2x = 5$$

$$\cancel{x^2 + 2^2} - 2x = 5$$

-10 Marks

we open this using

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

we will learn to use these later.

$$\boxed{4} \quad (a^m)^n = a^{m \times n}$$

POWER Ki POWER MULTIPLY HOGI.

$$\boxed{\text{Ex}} \quad 1) \quad (x^3)^2 = x^6$$

$\boxed{5}$ NEGATIVE POWERS

MATHS PPL HATE NEGATIVE POWERS.

$$a^{-m} = \frac{1}{a^m}$$

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

If we reciprocate base, +/- sign of power changes.

$$\underline{\text{Ex}} \quad 1) \quad 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$2) \quad \left(\frac{2}{3}\right)^{-2} = \left(\frac{3^{\cancel{2}}}{2^{\cancel{2}}}\right) = \frac{3^2}{2^2} = \frac{9}{4}$$

6

$$a^0 = 1$$

$$3^0 = 1$$

$$7^0 = 1$$

$$(-2)^0 = 1$$

0°
 → we do not like to talk about this.

Ex 1) $5^0 + 5^2 = 26$
 1 + 25

2) $4^0 - 4^{-2}$
 1 - $\frac{1}{4^2}$

$\frac{1^{1 \times 16}}{1 \times 16} - \frac{1}{16}$

$\frac{16 - 1}{16} = \frac{15}{16}$

3) $5^0 - 5^{-1}$
 1 - $\frac{1}{5^1}$

$\frac{1^{1 \times 5}}{1 \times 5} - \frac{1}{5}$

$\frac{5 - 1}{5} = \frac{4}{5}$

7

$$1^{\text{any}} = 1$$

$$1^3 = 1$$

$$1^5 = 1$$

$$1^{-3} = 1$$

$1^{3x-5} = 1$

Q:

$$\frac{3^2 \times 1^k}{2^3}$$

$\frac{9 \times 1}{8} = \frac{9}{8}$

8 FRACTIONAL POWERS (SURDS)

$$\sqrt{x} = x^{\frac{1}{2}}$$

Most fav power of
examiner.

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

It is compulsory to learn both methods.

Method 1	Method 2
1) $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^{\cancel{3} \times \frac{2}{3}} = 2^2 = 4$	1) $8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2 = (\sqrt[3]{8})^2 = (2)^2 = 4$
2) $64^{\frac{2}{3}} = (4^3)^{\frac{2}{3}} = 4^{\cancel{3} \times \frac{2}{3}} = 4^2 = 16$ $\swarrow \searrow$ $8^2 \quad 4^3$	2) $64^{\frac{2}{3}} = (64^{\frac{1}{3}})^2 = (\sqrt[3]{64})^2 = (4)^2 = 16$
3) $64^{\frac{3}{2}} = (8^2)^{\frac{3}{2}} = 8^{\cancel{2} \times \frac{3}{2}} = 8^3 = 512$	3) $64^{\frac{3}{2}} = (64^{\frac{1}{2}})^3 = (\sqrt{64})^3 = 8^3 = 512$

9

$$x^{\frac{a}{b}} = y \Rightarrow x = y^{\frac{b}{a}}$$

When power goes to other side of = sign,
it is reciprocated.

Ex 1) $x^{\frac{3}{2}} = 8$

$$x = 8^{\frac{2}{3}}$$

$$= (2^3)^{\frac{2}{3}} = 2^{\cancel{3} \times \frac{2}{3}} = 2^2 = 4$$

$$2) \quad x^2 = 16$$

$$x = \pm \sqrt{16}$$

$$x = (16)^{1/2}$$

V-IMP
10 POWERS AND DECIMALS ARE NOT FRIENDS.

$$\text{EX) 1) } (0.2)^{-2} = \left(\frac{2}{10}\right)^{-2} = \left(\frac{10}{2}\right)^2 = \frac{100}{4} = 25$$

$$2) \quad \sqrt[3]{0.008} = \sqrt[3]{\frac{8}{1000}} = \frac{\sqrt[3]{8}}{\sqrt[3]{1000}} = \frac{2}{10} = 0.2$$

EQUATION SOLVING

$$1) \quad 7^m \times 7^3 = 7^{12}$$

$$\cancel{7}^{m+3} = \cancel{7}^{12}$$

$$m+3 = 12$$

$$m = 9$$

You can cancel
base only if:

1) Single base (once)

2) Same base.

$$2) \quad \frac{5^m}{5^4} = 125$$

$$\cancel{5}^{m-4} = \cancel{5}^3$$

$$m-4 = 3$$

$$m = 7$$

$$3) \quad 7^m \times 7^4 = 1$$

$$7^{m+4} = 1$$

$$\cancel{7}^{m+4} = \cancel{7}^0$$

$$m+4 = 0$$

$$m = -4$$

We know	
$a^0 = 1$	
Desired base = 7	
$7^0 = 1$	

$$4) \quad 8^b = 2$$

$$(2^3)^b = 2$$

$$\cancel{2}^{3b} = \cancel{2}^1$$

$$3b = 1$$

$$b = \frac{1}{3}$$

When there is no power written on something, the power is always 1.

$$5) \text{ Simplify } \left(\frac{18 x^2 y}{6 x y^5} \right)^{-2}$$

$$\left(\frac{\cancel{3} \cancel{18}^1 x^2 \cancel{y}}{\cancel{6} \cancel{x} y^{\cancel{5}^4}} \right)^{-2}$$

$$\frac{y}{y^5} = \frac{\cancel{y}}{y y y y \cancel{y}} = \frac{1}{y^4}$$

$$\left(\frac{3x}{y^4} \right)^{-2} = \left(\frac{y^4}{3x} \right)^2$$

$$= \frac{(y^4)^2}{(3)^2 (x)^2}$$

$$= \frac{y^8}{9x^2}$$

$$6) \left(\frac{16 x^7 y^2}{x^3 y^{10}} \right)^{\frac{1}{2}}$$

$$\left(\frac{16 x^{\cancel{7}^4} y^{\cancel{2}^2}}{\cancel{x^3}^1 y^{\cancel{10}^8}} \right)^{\frac{1}{2}}$$

$$\left(\frac{16 x^4}{y^8} \right)^{\frac{1}{2}} = \frac{(16)^{\frac{1}{2}} (x^4)^{\frac{1}{2}}}{(y^8)^{\frac{1}{2}}}$$

$$= \frac{(4)^{\frac{1}{2}} (x^{\frac{2}{2}})^{\frac{1}{2}}}{(y^{\frac{8}{2}})^{\frac{1}{2}}}$$

$$= \frac{4 x^2}{y^4}$$

Expansions

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Factorize

$$a^2 - b^2 = (a+b)(a-b)$$

MOST IMPORTANT PAGE IN MATHS!

EXPANSIONS

Indices:

You are not allowed to open power directly if terms are being added/subtracted

Firstly:

$$(a+b)^2 = \cancel{a^2} + \cancel{b^2}$$
$$(a-b)^2 = \cancel{a^2} - \cancel{b^2}$$
$$(a \times b)^2 = a^2 \times b^2 \checkmark$$
$$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2} \checkmark$$

So, Expand:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(x+3)^2 = (x)^2 + 2(x)(3) + (3)^2$$
$$= x^2 + 6x + 9$$

$$(2x+5)^2 = (2x)^2 + 2(2x)(5) + (5)^2$$
$$= 4x^2 + 20x + 25$$

$$(3+4x)^2 = (3)^2 + 2(3)(4x) + (4x)^2$$
$$= 9 + 24x + 16x^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(x-3)^2 = (x)^2 - 2(x)(3) + (3)^2$$
$$= x^2 - 6x + 9$$

$$(2x-5)^2 = (2x)^2 - 2(2x)(5) + (5)^2$$
$$= 4x^2 - 20x + 25$$

FACTORIZATION

Ans: () ()

exp →

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Factorize

$$a^2 - b^2 = (a+b)(a-b)$$

TWO TERMS

Step 1: Take common if possible
Step 2: $a^2 - b^2 = (a+b)(a-b)$

FACTORIZE

1 $4x - 10$
 $2(2x - 5)$

2 $x^2 - 9$
 $(x)^2 - (3)^2$
 $(x+3)(x-3)$

3 $1 - 16x^2$
 $(1)^2 - (4x)^2$
 $(1+4x)(1-4x)$

4 $5x^2 - 20$
 $5[x^2 - 4]$

$5[(x)^2 - (2)^2]$
 $5[(x+2)(x-2)]$

5 $7x^2 - 63$
 $7[x^2 - 9]$
 $7[(x)^2 - (3)^2]$
 $7(x+3)(x-3)$

6 $4xy^3 + 16x^3y$
 $4xy[y^2 + 4x^2]$

This sign has to be -ve for $a^2 - b^2 = (a+b)(a-b)$ to be applied.

THREE TERMS: MIDDLE TERM BREAKING

1 $x^2 + 7x + 12$
 $x^2 + 3x + 4x + 12$
 $x(x+3) + 4(x+3)$
 $(x+3)(x+4)$

Sub	12	Add
11	1x12	13
4	2x6	8
1	3x4	7
		3+4=7

2 $x^2 + 6x - 16$

$x^2 + 8x - 2x - 16$

$x(x+8) - 2(x+8)$

$(x+8)(x-2)$

Sub	16	Add
15	1x16	17
6	2x8	10
0	4x4	8

$8-2=6$

Rule: If the first term of any group is negative, it is compulsory to take -ve common. When we take -ve common, signs inside bracket change.

3 $x^2 - 12x + 20$

$x^2 - 2x - 10x + 20$

$x(x-2) - 10(x-2)$

$(x-2)(x-10)$

Sub	20	Add
19	1x20	21
8	2x10	12
1	4x5	9

$2+10=12$

$-2-10=-12$

4

$$① x^2 - 2x - ② 24$$

$$x^2 - 6x + 4x - 24$$

$$x(x-6) + 4(x-6)$$

$$(x-6)(x+4)$$

Sub

24

Add

$$1 \times 24$$

$$2 \times 12$$

$$3 \times 8$$

②

$$4 \times 6$$

$$6 - 4 = 2$$

$$-6 + 4 = -2$$

5

$$2x^2 + 29x - 15$$

30

1x30

$$2x^2 + 30x - 1x - 15$$

$$2x(x+15) - 1(x+15)$$

$$(2x-1)(x+15)$$

6

$$x^2 - 4xy - 12y^2$$

$$x^2 - 6xy + 2xy - 12y^2$$

$$x(x-6y) + 2y(x-6y)$$

$$(x+2y)(x-6y)$$

SUB

12

ADD

1x12

4

2x6

3x4

$$6-2=4$$

$$-6+2=-4$$

FOUR TERMS Group and take
Common.

Ex

$$ax + ay - bx - by$$

$$a(x+y) - b(x+y)$$

$$(a-b)(x+y)$$

Sometimes there is no common
in the groups. We have to
rearrange the terms.

while rearranging always
keep both positive terms first
and then both negative term.

FACTORIZE

Ans () ()

Factorize:

$$\textcircled{1} x^2 - 7x + 12$$

$$x^2 - 3x - 4x + 12$$

$$x(x-3) - 4(x-3)$$

$$(x-4)(x-3)$$

12

1×12
 2×6
 3×4
 $3+4=7$
 $-3-4=-7$

SOLVE

$x = \square, x = \square$

SOLVE

$$x^2 - 7x + 12 = 0$$

$$x^2 - 3x - 4x + 12 = 0$$

$$x(x-3) - 4(x-3) = 0$$

$$(x-4)(x-3) = 0$$

12

1×12
 2×6
 3×4
 $3+4=7$
 $-3-4=-7$

Either: $x-4=0$, OR $x-3=0$

$$x=4$$

$$x=3$$

V. TRICKY

Q. Solve. $(2x+3)(x-1) = 0$ (1 Mark)

Either: $2x+3=0$

OR: $x-1=0$

$$2x = -3$$

$$x = \frac{-3}{2}$$

$$x = 1$$

IMP

$$(x+3)(x-5) = 2$$

Either: ~~$x+3=2$~~

OR: ~~$x-5=2$~~

In this question first expand, bring 2 to left side, then factorize again.

$$\begin{aligned} (x+3)(x-5) &= 2 \\ x^2 - 5x + 3x - 15 &= 2 \\ x^2 - 2x - 15 - 2 &= 0 \\ x^2 - 2x - 17 &= 0 \end{aligned}$$

Now the exam question will be solved using factorization.

QUADRATIC EQUATION SOLVING

$$ax^2 + bx + c = 0$$

FACTORIZATION	COMPLETED SQUARE	FORMULA.
Two terms:	$a(x-b)^2 + c$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
1) Common		
2) $a^2 - b^2 = (a+b)(a-b)$	$\rightarrow a, b \text{ and } c \text{ are numbers}$	$\rightarrow \text{Make sure you write formula and do detailed working to earn 4 Marks.}$
Three term:	$\rightarrow +/- \text{ signs can be different.}$	
Middle Term Break	$2(x+3)^2 + 5$	
Four Terms:	$3(x-1)^2 - 2$	
Group and take common	$-2(x+5)^2 + 5$	
2-3 Marks	Only use completed square method when the first part of question requires to go to this form.	3-4 Marks
No mention of decimal places or significant figures		Question has specified decimal places or significant figures.

Q: Solve by factorization

(a) $x^2 - 6x - 16 = 0$

$\textcircled{1}x^2 - 6x - \textcircled{16} = 0$

$x^2 - 8x + 2x - 16 = 0$

$x(x-8) + 2(x-8) = 0$

$(x+2)(x-8) = 0$

Either: $x+2=0$

$x = -2$

OR: $x-8=0$

$x = 8$

$\boxed{16}$

1×16

$\textcircled{2 \times 8}$

4×4

$8-2=6$

$-8+2=-6$

2) $x^2 - 25 = 0$

$(x)^2 - (5)^2 = 0$

$(x+5)(x-5) = 0$

$x+5=0$, $x-5=0$

$x = -5$

, $x = 5$

Another approach:

$x^2 - 25 = 0$

$x^2 = 25$

$x = \pm\sqrt{25}$

$x = \pm 5$

$x=5$, $x=-5$

Q. Express $2x^2 - 12x + 10$ in form $a(x-b)^2 + c$
completed square form.

$$2x^2 - 12x + 10$$

$$2[x^2 - 6x + (3)^2 - (3)^2 + 5]$$

$$2[(x-3)^2 - 9 + 5]$$

$$2[(x-3)^2 - 4]$$

$$\begin{array}{c} 2(x-3)^2 - 8 \\ a(x-b)^2 + c \end{array}$$

$$\begin{array}{ccc} a=2 & -b=-3 & +c=-8 \\ & b=3 & c=-8 \end{array}$$

Q. Express $3x^2 + 24x + 30$ in form $a(x-b)^2 + c$
completed square form.

$$3x^2 + 24x + 30$$

$$3 \left[x^2 + 8x + (4)^2 - (4)^2 + 10 \right]$$

$$3 \left[(x + 4)^2 - 16 + 10 \right]$$

$$3 \left[(x + 4)^2 - 6 \right]$$

$$\begin{array}{c} 3(x+4)^2 - 18 \\ a(x-b)^2 + c \end{array}$$

$$a=3 \quad -b=+4 \quad +c=-18$$

$$b=-4 \quad c=-18$$

Q: Express $2x^2 - 20x + 7$ in form $a(x-b)^2 + c$
completed square form.

$$2x^2 - 20x + 7$$

$$2 \left[x^2 - 10x + (5)^2 - (5)^2 + \frac{7}{2} \right]$$

$$2 \left[(x-5)^2 - 25 + \frac{7}{2} \right]$$

$$2(x-5)^2 - 50 + 7$$

$$\begin{array}{r} 2 \times \frac{7}{2} \\ = 7 \end{array}$$

$$2(x-5)^2 - 43$$

$$a(x-b)^2 + c$$

$$a=2, \quad -b=-5, \quad +c=-43$$

$$b=5, \quad c=-43$$

Q: (i) Express $x^2 - 6x + 3$ in form $\underbrace{(x-a)^2 - b}_{\text{completed square form.}}$

$$\begin{aligned} & \xrightarrow{\text{Half}} x^2 - 6x + (3)^2 - (3)^2 + 3 \\ & (x-3)^2 - 9 + 3 \\ & (x-3)^2 - 6 \end{aligned}$$

$$\begin{aligned} x^2 - 6x + 3 & \longrightarrow (x-3)^2 - 6 \\ & (x-a)^2 - b \\ & -a = -3, \quad -b = -6 \\ & a = 3, \quad b = 6 \end{aligned}$$

(ii) Hence solve $x^2 - 6x + 3 = 0$ giving your answer in form $p \pm \sqrt{q}$ stating values of p and q .

$$\begin{aligned} x^2 - 6x + 3 &= 0 \\ (x-3)^2 - 6 &= 0 \\ (x-3)^2 &= 6 \end{aligned}$$

Take square root on both sides.

$$\begin{aligned} x-3 &= \pm \sqrt{6} \\ x &= 3 \pm \sqrt{6} \\ & p \pm \sqrt{q} \end{aligned}$$

$$p=3, \quad q=6$$

Note for stupid:

Square remove karain
toh square root ayega
+/- k sath.

QUADRATIC FORMULA

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Q. Solve $2x^2 - 3x - 1 = 0$ up to 2dp. (4)
 $a=2$, $b=-3$, $c=-1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

↗ This is the only formula you have to write.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

$$x = \frac{3 + \sqrt{17}}{4}$$

or

$$x = \frac{3 - \sqrt{17}}{4}$$

$$3 + \sqrt{17} \div 4 = \text{wrong}$$

$$(3 + \sqrt{17}) \div 4 = \text{correct}$$

$$x = 1.78$$

$$x = -0.28$$

now having 2dp is compulsory.

MAKING SUBJECT OF FORMULA

TRICK: Term being made subject should be positive.

Q. $y = \frac{3 - 2x}{5}$, Make x subject.

$$5y = 3 - 2x$$

$$5y + 2x = 3$$

$$2x = 3 - 5y$$

$$x = \frac{3 - 5y}{2}$$

Make k the subject of the formula

$$\sqrt{\frac{h}{k}} = 3.$$

[2]

SQUARE BOTH SIDES

$$\left(\sqrt{\frac{h}{k}}\right)^2 = (3)^2$$

$$\frac{h}{k} = 9$$

$$h = 9k$$

$$k = \frac{h}{9}$$

$$\sqrt{\frac{h}{k}} = 3$$

$$\left(\frac{h}{k}\right)^{\frac{1}{2}} = 3$$

$$\frac{h}{k} = 3^2$$

$$h = 9k$$

$$\frac{h}{9} = k$$

Given that $T = 2\pi\sqrt{\frac{L}{g}}$,

express g in terms of π , T and L .

[3]

4024/02/O/N/06

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{T}{2\pi} = \left(\frac{L}{g}\right)^{\frac{1}{2}}$$

$$\left(\frac{T}{2\pi}\right)^2 = \frac{L}{g}$$

$$\frac{T^2}{4\pi^2} = \frac{L}{g}$$

$$gT^2 = 4\pi^2 L$$

$$g = \frac{4\pi^2 L}{T^2}$$

Make t the subject of the formula $p = \frac{4t+1}{2-t}$.

$$p(2-t) = 4t+1$$

$$2p-pt = 4t+1$$

$$2p-1 = 4t+pt$$

$$2p-1 = t(4+p)$$

$$t = \frac{2p-1}{4+p}$$

Answer [3]

(P1) 3Marks = Longest question of paper

(P2) 3Marks = Long question.

SIMULTANEOUS EQUATIONS

ELIMINATION

Q:
$$\begin{aligned} 2x + 3y &= 5 \\ x - 4y &= 12 \end{aligned}$$

Q:
$$\begin{aligned} 3x + y &= 2 \\ x + 2y &= 7 \end{aligned}$$

SUBSTITUTION.

Q:
$$\begin{aligned} x + 2y &= 3 \\ y &= 9x - 3 \end{aligned}$$

Q:
$$\begin{aligned} 3x + 5y &= 9 \\ 2x &= 3y. \end{aligned}$$

ELIMINATION

Solve the simultaneous equations

Make one term same
in both equations.

Make sure they have
opposite +/- signs.

$$\begin{aligned} (2x - y &= 16.) \times 2 \\ 3x + 2y &= 17. \end{aligned}$$

$$4x - 2y = 32$$

$$3x + 2y = 17$$

$$\hline 7x = 49$$

$$x = \frac{49}{7} = 7$$

Put this value in ANY
Eq. above.

$$3x + 2y = 17$$

$$3(7) + 2y = 17$$

$$21 + 2y = 17$$

$$2y = -4$$

$$y = -2$$

Answer $x = 7$

$y = -2$ [3]

Solve the simultaneous equations

lets make x-term
same this time:

to make +/-
signs opposite
change all signs
of bottom eq.

$$\begin{array}{l} 3 \times (2x - y = 16) \\ 2 \times (3x + 2y = 17) \end{array}$$

$$\begin{array}{r} 6x - 3y = 48 \\ -6x + 4y = -34 \\ \hline -7y = 14 \\ y = \frac{14}{-7} \\ y = -2 \end{array}$$

$$3x + 2y = 17$$

$$3x + 2(-2) = 17$$

$$3x - 4 = 17$$

$$3x = 21$$

$$x = \frac{21}{3}$$

$$x = 7$$

Answer $x = 7$

$y = -2$ [3]

SUBSTITUTION

Solve the simultaneous equations

$$x + y = 29,$$

$$4x = 95 - 2y.$$

$$\longrightarrow x = 29 - y$$

Make any alphabet
subject of equation

$$4x = 95 - 2y$$

$$4(29 - y) = 95 - 2y$$

$$116 - 4y = 95 - 2y$$

$$116 - 95 = 4y - 2y$$

$$21 = 2y$$

$$y = \frac{21}{2}$$

$$\begin{aligned} x &= 29 - y \\ &= 29 - \frac{21}{2} \\ &= \frac{58 - 21}{2} \end{aligned}$$

$$x = \frac{37}{2}$$

Solve the simultaneous equations

$$12 = \frac{35y}{3} - 1$$

you cannot
cross multiply.

$$3x = 7y, \longrightarrow x = \frac{7y}{3}$$

$$12y = 5x - 1$$

$$12y = \frac{5}{1} \left(\frac{7y}{3} \right) - 1$$

$$12y + 1 = \frac{35y}{3}$$

$$36y + 3 = 35y$$

$$36y - 35y = -3$$

$$y = -3$$

$$x = \frac{7y}{3} = \frac{7(-3)}{3}$$

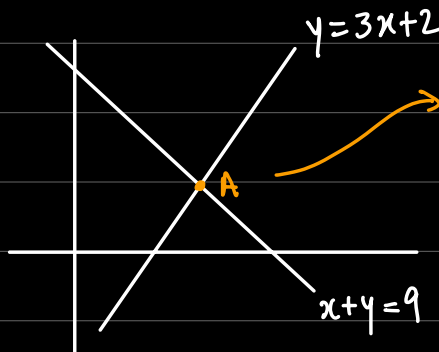
$$x = -7$$

Answer $x = \dots\dots\dots$

$y = \dots\dots\dots$ [3]

WHAT DOES SOLVING SIMULTANEOUS EQUATIONS DO?

It always gives us POINT OF INTERSECTION of any two graphs.



To find coordinates of A
you have to solve
simultaneous eq for
both.

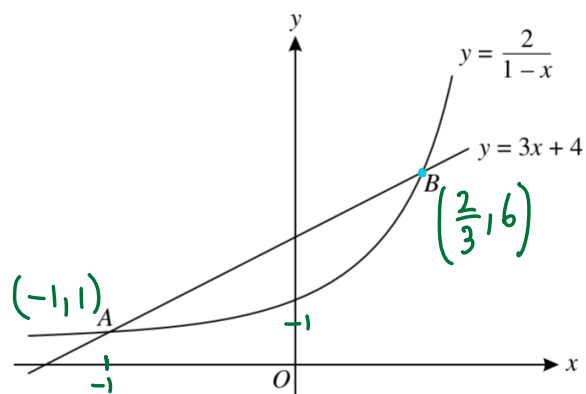
SIMULTANEOUS EQUATION : 1) ELIMINATION

2) SUBSTITUTION

(SIR ZAIN'S
Method)

3) Make y subject in both equations and

put them equal (This method may get you
in trouble if you have less space)



The diagram shows part of the curve $y = \frac{2}{1-x}$ and the line $y = 3x + 4$. The curve and the line meet at points A and B.

- (i) Find the coordinates of A and B. (POINTS OF INTERSECTION) (SIMULT.) (6 marks)

$$y = \frac{2}{1-x}$$

$$y = 3x + 4$$

$$\frac{2}{1-x} = 3x + 4$$

$$2 = (3x + 4)(1 - x)$$

$$2 = 3x - 3x^2 + 4 - 4x$$

Bring everything to left side.

$$3x^2 + 4x - 3x + 2 - 4 = 0$$

$$3x^2 + x - 2 = 0$$

$$3x^2 + 3x - 2x - 2 = 0$$

$$3x(x+1) - 2(x+1) = 0$$

$$(3x - 2)(x + 1) = 0$$

$$3x - 2 = 0$$

$$3x = 2$$

$$x = \frac{2}{3}$$

$$x + 1 = 0$$

$$x = -1$$

6

1x6

3x2

3-2=1

3

$$y = 3x + 4$$

$$y = 3\left(\frac{2}{3}\right) + 4 = 6$$

$$\left(\frac{2}{3}, 6\right)$$

$$y = 3x + 4$$

$$y = 3(-1) + 4 = 1$$

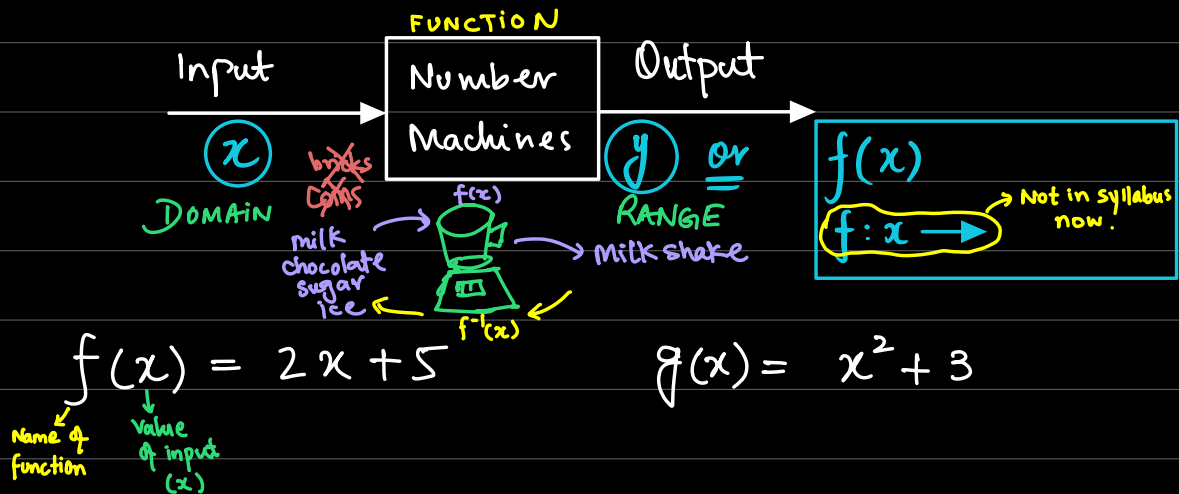
$$(-1, 1)$$

FUNCTIONS

(4 MARKS) (P1)

In old syllabus Functions was different for OLEVELS (4024) and IGCSE (0580).

From June 2025 onwards its same for both.



$$1) f(3) = 2(3) + 5 = 11$$

$$5) g(t) = t^2 + 3$$

$$2) g(2) = (2)^2 + 3 = 7$$

$$6) f(k) = 2k + 5$$

$$3) f(10) = 2(10) + 5 = 25$$

$$7) f(t-3) = 2(t-3) + 5$$

Expand later.

$$4) g(5) = (5)^2 + 3 = 28$$

$$8) g(k+1) = (k+1)^2 + 3$$

Expand later.

Type 1

Q:

$$f(x) = 2x + 5$$

Given that $f(3) = t + 2$, find t .

$$f(3) = t + 2$$

$$2(3) + 5 = t + 2$$

$$11 = t + 2$$

$$\boxed{9 = t}$$

Q:

$$g(x) = 3x - 5$$

Given that $g(k) = 12$, find k .

$$g(k) = 12$$

$$3k - 5 = 12$$

$$3k = 17$$

$$k = \frac{17}{3}$$

Q.

$$f(x) = 4x - 3$$

Given that $f(t) = t$, find t .

$$f(t) = t$$

$$4t - 3 = t$$

$$4t = 3 + t$$

$$3t = 3$$

$$t = 1$$

Q.

$$g(x) = 2x - 3$$

Given that $g(k) = k$, find k .

$$g(k) = k$$

$$2k - 3 = k$$

$$2k = k + 3$$

$$2k - k = 3$$

$$k = 3$$

Q.

$$f(x) = 3x + 7$$

Given that $f(3t+1) = 12t$ find t .

$$f(3t+1) = 12t$$

$$3(3t+1) + 7 = 12t$$

$$9t + 3 + 7 = 12t$$

$$10 = 3t$$

$$t = \frac{10}{3}$$

INVERSE OF A FUNCTION

Symbol: $f^{-1}(x)$

Beware: f^{-1} means
put $x = -1$ in function
 $f(x)$

STEPS:

1) Replace $f(x) \rightarrow y$

2) Make x subject

3) Replace $x \rightarrow f^{-1}(x)$
 $y \rightarrow x$

Q. $f(x) = 2x - 3$

Find inverse.

$$y = 2x - 3$$

$$y + 3 = 2x$$

$$x = \frac{y + 3}{2}$$

$$f^{-1}(x) = \frac{x + 3}{2}$$

Q. $f(x) = 12 - 2x$, Find inverse.

$$\begin{array}{l|l} y = 12 - 2x & f^{-1}(2) = \frac{12-2}{2} \\ y + 2x = 12 & = 5 \\ 2x = 12 - y & \\ x = \frac{12-y}{2} & \\ f^{-1}(x) = \frac{12-x}{2} & \end{array}$$

Q. $f(x) = \frac{2x+3}{5x}$ Find inverse.

$$\begin{array}{l} y = \frac{2x+3}{5x} \\ 5xy = 2x+3 \\ 5xy - 2x = 3 \\ x(5y-2) = 3 \\ x = \frac{3}{5y-2} \\ f^{-1}(x) = \frac{3}{5x-2} \end{array} \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{1 mark (M1)} \\ \\ \\ \\ \end{array}$$

$$\left. \begin{array}{l} f^{-1}(x) = \frac{3}{5x-2} \end{array} \right\} \text{1 mark (A1)}$$

Type 3 Composite Functions. (Function within a function)

O-Levels (4024)
(Not for Oct 2024)
(only for June 2025 onward)

IGCSE (0580)
Included for both
Oct 2024 and June 2025

Q: $f(x) = 2x + 3$ $g(x) = 5x^2 + 6$

Step 1: Write first function with () instead of x .

Step 2: Put the value of second function in ().

$$\begin{aligned}\text{(i)} \quad fg(x) &= 2(5x^2 + 6) + 3 \\ &= 10x^2 + 12 + 3 \\ fg(x) &= 10x^2 + 15\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad gf(x) &= 5(2x + 3)^2 + 6 \\ &= 5[(2x)^2 + 2(2x)(3) + (3)^2] + 6 \\ &= 5[4x^2 + 12x + 9] + 6 \\ &= 20x^2 + 60x + 45 + 6 \\ gf(x) &= 20x^2 + 60x + 51\end{aligned}$$

Q: $f(x) = 2x + 1$ $g(x) = (3x + 5)^2$

(i) $fg(x) = 2((3x+5)^2) + 1$

(ii) $gf(x) = (3(2x+1) + 5)^2$

(iii) $ff(x) = 2(2x+1) + 1$

(iv) $gg(x) = (3((3x+5)^2) + 5)^2$

19 $f(x) = 5x + 2$ $g(x) = x^2 - 5$ $h(x) = 7 - x$.

(a) Find $f(3)$. $= 5(3) + 2 = 17$

..... 17 [1]

(b) Find $gf(x)$.

Give your answer in the form $ax^2 + bx + c$.

$$\begin{aligned} gf(x) &= (5x + 2)^2 - 5 \\ &= (5x)^2 + 2(5x)(2) + (2)^2 - 5 \\ &= 25x^2 + 20x + 4 - 5 \end{aligned}$$

$$gf(x) = 25x^2 + 20x - 1$$

..... [3]

(c) Solve $\frac{3}{hf(x)} = -1$.

$$\frac{3}{7 - (5x + 2)} = -1$$

$$\begin{aligned} \frac{3}{7 - 5x - 2} &= -1 \\ \frac{3}{5 - 5x} &= -1 \end{aligned}$$

$$3 = -1(5 - 5x)$$

$$3 = -5 + 5x$$

$$8 = 5x$$

$$x = \frac{8}{5}$$

$x =$ [3]